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Birkbeck Working Papers in Economics & Finance

ISSN 1745-8587

Birkbeck
UNIVERSITY OF LONDON

School of Economics, Mathematics and Statistics

BWPEF 0509

Estimation of the Risk Attitude of the Representative UK Pension Fund Investor

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June 2005

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Estimation of the Risk Attitude of the Representative UK Pension Fund Investor

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Abstract: The purpose of this paper is to use UK pension funds asset allocation information to model the risk attitude of the representative UK pension fund investor. Unlike the previous literature on loss aversion, we find that UK pension funds display risk aversion with respect to gains and to losses. Such a finding suggests a greater degree of responsibility by UK pension funds that they are usually credited with.

Keywords: LA Utility Function, Non-linear Regression, LAD, UK pension fund

Acknowledgement: Wei Xia would like to thank Birkbeck College for the generous funding support and is grateful to Mr. Piergiorgio Alessandri, Dr. Pedro Bacao and Dr. Young-Bae Kim for their helpful comments

1 Introduction

The estimation of parameters in the Loss Aversion (**LA**) utility function has attracted attention from many scholars since the LA utility function was first put forward by Kahneman and Tversky (**KT**) (1979, 1992). Inter Alia, Barberis, Huang and Santos (2001) have addressed the asset allocation problem for a loss averse investor in a one period world. Berkelaar and Kouwenberg (2000a, 2000b) studied how risk aversion could affect the investment decisions. More recently, Hwang and Satchell (2003) have shown how to investigate admissible ranges of the parameters values in the LA utility function using the one period asset allocation decision model; they also carried out an empirical calibration study into the LA utility function parameters based on UK and US investment data. However, their methodology failed to identify unique parameters. The purpose of this paper is to find unique parameters using UK pension funds investment data; we shall achieve this by estimating investor demands, rather than the calibration methods previously employed.

There are also a number of papers about UK pension fund investor. Among them, Tonks (1999) provided a comprehensive review on the pensions policy in the UK. Blake, Lehmann, and Timmermann (1999) showed that the UK pension fund industry are dominated by five largest funds, and unlike the US pension fund industry, UK pension funds have substantial disincentives to manage portfolio actively in ways that risk large differences in relative performance. "The peer group is concerned with maintaining its reputation for services and reliable, if similar and unspectacular, performance, the structure one would expect if there were no ex ante differences in performance ability." The cross-sectional variation in average ex post returns is also surprisingly small.

In section 2, we briefly review the one period optimal investment structure decision model derived in Hwang and Satchell (2003). Next we discuss the inference of LA parameters of UK pension funds by running a nonlinear regression based on the above model. We carried out the estimation with the Least Absolute Deviation (**LAD**) method, which we implemented by numerical search. In section 3 and section 4, we will show how we use the UK pension funds data and financial market data in our regression and how we run the regression. In section 5, the results we get from the regressions are presented, which also considers a comparison to the results in Hwang and Satchell (2003). We present our conclusions in section 6. In contrast to the traditional experimental finding that most investors are risk averse to gain but risk loving to loss, we find that UK pension fund investors are extremely risk averse to both gain and loss. Since there has been very little estimation of loss-averse demands, we believe our result to be of some interest.

2 Theoretical Framework

In this section, we present the notation and assumptions used in this paper, and give a brief review of the theoretical framework in Hwang and Satchell (2003). In the second subsection, we analyze econometric problems and propose possible solutions.

2.1 Some notation and assumptions

The following notation will be used in this paper.

2.1.1 Notation in this paper:

W_0 = Initial Wealth of investor

W_1 = Final Wealth of investor after one period

B = Benchmark

$X = W - B$

θ = Proportion invested in risky asset (i.e. in equity)

v_1 = Parameter in KT LA utility function for X⩾0

v_2 = Parameter in KT LA utility function for X<0

λ = Parameter in KT LA utility function for X<0

P = Probability that equity beats bond i.e. Probability(X>0)

ε = Regression Residuals

2.1.2 Assumptions in our discussion:

1) We assume there are only two assets in the world, i.e. a risky asset with rate of return R for one period and a risk-free asset with return R_f. Define excess return y = R- R_f, and P=Probability(y>0).

2) We assume KT Loss Aversion utility function as our utility function for the agent.

$$\begin{aligned} u(X) &= \frac{X^{v_1}}{v_1}, \text{ if } X \geq 0 \\ &= -\lambda \frac{(-X)^{v_2}}{v_2}, \text{ if } X < 0 \end{aligned} \tag{1.1}$$

where v_1>0, v_2>0, λ>0

Therefore, we have following immediate implications:

$$v_1 \begin{cases} (0,1), \Rightarrow \text{ Agent is Risk Averse to Gain} \\ =1, \Rightarrow \text{ Agent is Risk Neutral to Gain} \\ >1, \Rightarrow \text{ Agent is Risk Loving to Gain} \end{cases} \text{ and}$$

$$v_2 \begin{cases} (0,1), \Rightarrow \text{ Agent is Risk Loving to Loss} \\ =1, \Rightarrow \text{ Agent is Risk Neutral to Loss} \\ >1, \Rightarrow \text{ Agent is Risk Averse to Loss} \end{cases}$$

while the larger the λ, the more severe the loss aversion.

3) We assume that the empirical investment structure is the expected optimal structure that UK pension fund investors hold, i.e. we equate the theoretical and empirical asset allocation values. Moreover, we assume that UK pension fund investors follow myopic behaviour to be consistent with our one period model.

2.2 Brief Review of the theoretical framework in Hwang and Satchell (2003)

We show how we derive our optimal investment decision model as follows:

$$\begin{aligned} u(W_1 - B) &= u_1(W_1 - B),\ if\ X \geq 0\ i.e.\ W_1 \geq B \\ &= -\lambda u_2(W_1 - B),\ if\ X < 0\ i.e.\ W_1 < B \Rightarrow \end{aligned}$$

The Loss Aversion Utility function of the agent is:

$$U_{LA} = \frac{1}{v_1}\left(\theta W_0\right)^{v_1} pu^+ - \frac{\lambda}{v_2}\left(\theta W_0\right)^{v_2}\left(1-p\right)u^- \tag{1.2}$$

where $u^+ = E(y^{v_1} \mid y > 0)\ and\ u^- = E((-y)^{v_2} \mid y < 0)$

Take first derivative with respect to θ to maximize the $U_{LA} \Leftarrow$

$$\left.\begin{aligned} &U_{LA}^{'} = W_0^{v_1} p\theta^{v_1-1}u^+ - \lambda W_0^{v_2}(1-p)\theta^{v_2-1}u^- \\ &if\ v_1 \neq v_2 \end{aligned}\right\} \Rightarrow \tag{1.3}$$

$$\left.\begin{array}{l}\Rightarrow \theta = \dfrac{1}{W_0}\left(\dfrac{u^+ p}{\lambda u^-(1-p)}\right)^{\frac{1}{v_2 - v_1}} \\ Since\ 0<\theta<1\end{array}\right\} \Rightarrow \qquad (1.4)$$

$$\left.\begin{array}{l}\Rightarrow \ln(\theta) = -\ln(W_0) + (\dfrac{1}{v_2 - v_1})\ln(u^+ p) - \dfrac{1}{v_2 - v_1}\ln(\lambda u^-(1\text{-}p)) \\ \text{Since } u(.) \text{ have CARA property, setting } W_0 = 1\end{array}\right\} \Rightarrow$$

$$\Rightarrow \ln(\theta) = \frac{1}{v_2 - v_1}[\ln(u^+) + \ln(p) \text{-} \ln(\lambda) \text{-} \ln(u^-) \text{-} \ln(1\text{-}p)] \qquad (1.5)$$

where $u^+ = E(y^{v_1} \mid y > 0)\ and\ u^- = E((-y)^{v_2} \mid y < 0)$

Equation (1.5) is the starting point of this paper. At this point, setting W = 1 is an innocuous, however, it will have repercussions in time series regression. We shall infer the LA parameters by running a regression on equation (1.5). For detail see Hwang and Satchell (2003).

2.3 Estimation problems and possible solutions

Establishing the regression as following:

$$\ln(\theta_i) = \frac{1}{v_2 - v_1}[\ln(u_i^+) + \ln(p) \text{-} \ln(\lambda) \text{-} \ln(u_i^-) \text{-} \ln(1\text{-}p)] + \varepsilon_i \qquad (1.6)$$

where $u_i^+ = E(y^{v_1} \mid y > 0)\ and\ u_i^- = E((-y)^{v_2} \mid y < 0)$, i stands for i^{th} sample observation (time). v_1, v_2 and λ are the target parameters to be estimated.

We assume that mean of ε_i is zero, and the usual assumptions of regression apply.

2.3.1 Difficulties of the parameter estimation

We encountered two difficulties in running a regression on equation (1.6):

1) Two terms in the regression are not linear.

Since $u^+ = E(y^{v_1} \mid y > 0)\ and\ u^- = E((-y)^{v_2} \mid y < 0)$, so we need to run a nonlinear regression to get the estimation.

2) Not only is it a nonlinear regression but also we can hardly work out an explicit closed form of u^+ and u^-, since these two terms are conditional expectations of a power function and the numbers of y > 0 and y < 0 out of each sample observations are uncertain for a closed form. Therefore, it is also very difficult to linearize the regression.

2.3.2 Possible solutions

1) In Hwang and Satchell (2003), the authors used the Knight, Satchell and Tran (**KST**) distribution and historical return data to get an approximation for u^+ and u^- by calculating an integral, and they derived an explicit closed form.

2) In this paper we solve these two problems in a different way. Although we cannot get an explicit closed form of u^+ and u^- for our sample observations, yet we do have some idea about the possible range of v_1 and v_2 from other scholars' research, which is positive but not too large numbers. Therefore, at the first step we can feed in a combination of v_1, v_2 and λ from a grid to calculate a fitted value of θ, $\hat{\theta}$. We can compute a residual ε for each sample observation as well as the sum of absolute residuals, $\sum_{i=1}^{n}|\varepsilon_i|$, ($n$ is the number of observation) for this particular trial. Next we try all possible combinations of v_1, v_2 and λ by simulation in a particular range and degree of accuracy to minimize the sum of absolute residuals, $\sum_{i=1}^{n}|\varepsilon_i|$, and then take the combination of v_1, v_2 and λ that produces the minimum sum of absolute residuals as our best estimation. This approach is a nonlinear Least Absolute Deviation (LAD) method. We minimize the sum of absolute residuals instead of squared residuals, because if not, it will make the outliers dominate the result.

Processing the regression as following:

$$\varepsilon_i = \ln(\theta_i) - \frac{1}{v_2 - v_1}\left[\ln(u_i^+) + \ln(p) - \ln(\lambda) - \ln(u_i^-) - \ln(1-p)\right] \quad (1.7)$$

where $u_i^+ = E(y^{v_1} \mid y > 0)$ *and* $u_i^- = E((-y)^{v_2} \mid y < 0)$, i stands for i^{th} sample observation (time). For conditions of validity in LAD nonlinear estimation and the asymptotic properties of LAD estimator see Amemiya (1982), for robust property of LAD estimator see Powell (1984) and for the performance of LAD based estimator on small and medium sample see Mishra and Dasgupta (2004). We also need the real data surface of $\sum_{i=1}^{n} |\varepsilon_i| = f(v_1, v_2, \lambda)$ without local jumps to make our estimation valid.

The formal estimation problem is $\underset{v_1, v_2, \lambda}{Min}\left(\sum_{i=1}^{n} |\varepsilon_i|\right)$, where equation 1.7 defines residual.

3 Data Collection and Handling

3.1 Data Collection

We collected the following data:

UK pension fund average investment structure data from 1962 to 2000 (Data Source: National Statistics, WM)

The data are attached in **Appendix A**. There are seven investment categories in the table. The portion in UK equity fluctuates around 50% in our sample period, but after a peak in 1972 and a floor in 1974, it decreases down to 44% in 1970s before its rebound in the 1980s and 1990s. The fraction invested in Overseas Equity goes up gradually from 0% in 1965 to over 20% in 1990s, while UK bonds investment decreases steadily from 50% in 1960s to less than 10% in 1990s. UK pension fund investors began their Index-linked Gilt and Overseas Bonds investment in 1980s, but they keep this part of investment within 10% of their total stake. The portion in Cash rises from 2% after 1972 and slumps down to 4% again in early 1980s. After that, it fluctuates between 4% to 7% from late 1980s. The fraction in Property goes up almost steadily from 2% in 1965 to 19% in 1975 and then stays around 17% from 1974 to 1981 before its decrease in the 1980s and 1990s. We shall treat this data set as the average of the UK pension investment, including both pooled and segregated funds. We shall also include the asset allocation from 1990 to 2000 for the WM 2000 universe, which consists of most larger UK segregated pension funds; this has a number of different characteristics from the national average, there being more equity investment and less property investment.

We can clearly identify some economic events from the data. Among them, it is notable that in the early 1970s the collapse of the Breton Wood System and the dramatic rise of oil prices depressed the world economy and made the global markets much more volatile than before. The portions invested in Equity and Bond dramatically decreased, while the fractions in Cash and Property increased, especially in 1974. The high inflation in the late 1970s also left its portrait in our data. In that period, UK pension fund investors reduced their exposure to Cash and held a record high level in Property to avoid depreciation.

To proxy asset class returns, we use the following

For equity, UK FTSE all shares index from 1962 to 2000 both annually and monthly.
For bond, UK 20 years Gilt bond yield data from 1964 to 2000 both annually and monthly.
For alternative bond, UK 3 months Government bill yield data from 1972 to 2000 both annually and monthly. (We tried to collect the data over the same period as that of UK 20 years Gilt bond yield data, but this is the maximum we can get. We assumed that the UK government bond or bill investment is the fixed income benchmark in our model. Data in the above are collected from Data Stream.)

3.2 Data Handling

In order to make the data consistent with our model, we need to adjust the raw data.
We reclassify the UK pension fund average investment structure data into only two categories, fixed income instrument and equity, to find an adjusted investment structure fraction, θ, for each observation. Since we do not have detailed sub-categories, we consider all bonds investment as fixed income assets. If the investment, such as Property, is difficult to tell whether it is equity or not, then we put it into the two categories evenly. Property can be decomposed into an equity component and a fixed income component. A great deal of calculation will be needed to find

appropriate weights for this decomposition, furthermore they would be time-varying; these considerations determine our choice above.

It should be noticed that by doing the reclassification, we ignore some effects of diversification of risk. Although individual international investment might have higher risk than in the domestic market, the financial literature argues that diversification across markets can reduce risk for the whole portfolio. However, we replace all other assets' returns with UK returns to be consistent with our theoretical model. Although we regard this as a sensible and acceptable approximation, readers should be aware of this.

We calculated the monthly and annual rate of return for FTSE all shares index RFT_{month_i}. Since bond yield rates by convention are quoted in annual basis, we need to convert the yield of 20 years Gilt bond yield and 3 months Government bill yield into monthly rate of return, $RGilt_{month_i}$ and $RBill_{month_i}$ respectively.

4 Some estimation details.

We need to construct time-varying u^+ and u^- for each year using monthly returns of equity and fixed income. Since u^+ and u^- are conditional expectations of annual returns, we construct them as following:

$$u^+ = E\left(\left(12y_{month_i}\right)^{v_1} \middle| y_{month_i} > 0\right)$$

$$u^- = E\left(\left(-12y_{month_i}\right)^{v_2} \middle| y_{month_i} < 0\right)$$

where $y_{month_i} = RFT_{month_i} - RBill_{month_i}$ with 3 months Government bill as Benchmark

$y_{month_i} = RFT_{month_i} - RGilt_{month_i}$ with 20 years Gilt bond as Benchmark to make u^+ and u^- be an implicit function of v_1 and v_2. Then given a pair of specific values of v_1 and v_2, we have a pair of values of u^+ and u^-, the term 12 y_{month_i} stands for annualized excess rate of return for month i.

Therefore, from equation (1.7) for annual observations, we can now calculate the fitted residual $\varepsilon_i = \ln(\theta_i) - \frac{1}{v_2 - v_1}\left[\ln(u_i^+) + \ln(p) - \ln(\lambda) - \ln(u_i^-) - \ln(1-p)\right]$ for a given set of v_1, v_2 and λ. Then we calculate the sum of absolute residuals ($\sum_{i=1}^{n}|\varepsilon_i|$).

We use numerical methods to try out all possible combinations of v_1, v_2 and λ with an appropriate degree of accuracy in a target value range. In our estimation, we made v_1 and v_2 ranging from 0.01 to 10 with a minimum increment of 0.01 while λ ranging freely from 0.01 to arbitrary large real number to get a converging result with a minimum increment of 0.01. Therefore, we can work out the optimal combination of v_1, v_2 and λ which produces the minimum sum of absolute residuals as our best estimation. We can also control λ to a hypothesized constant to test the optimal result of v_1 and v_2 conditional on that constant. The possible range and degree of accuracy for each parameter can be changed according to the user's needs.

5 Estimation Results and Analysis

We run the regression subject to the following parameter ranges and degrees of accuracy:

1) Run the regression with λ ranging from 0.01 to an arbitrary large real number with minimum increment of 0.01 to get a convergent result as the global optimal.

2) Run the regression with controlled λ on some particular values in order to learn the data pattern of $\sum_{i=1}^{n}|\varepsilon_i|$, v_1 and v_2 and to compare the results with the calibration results in Hwang and Satchell (2003). So we chose λ=1.5, 2.25, 3 respectively for the comparison.

3) Run the above regressions with the data of different benchmarks, that is 3 months UK Government bill with $\hat{P}$ = 0.611 over 1972--2000 and 20 years UK Gilt bond

with $\hat{P}$ = 0.621 over 1965--2000 respectively. The value of $\hat{P}$ comes from historical data we collected.

5.1 Results with 3 months UK Government bill as Benchmark.

Because of the constraint of available market data, we only have 29 observations (1972-2000) for this regression. The results are listed in **Table 1 in Appendix B.** This table shows the results conditional on different value of λ and the results with Bolded numbers are two local minimum, one of which has a small value of λ will be rejected according to the Hwang and Satchell's admissible ranges analysis, see Hwang and Satchell(2003).

$v_2 - v_1 > 0$ and $\lambda \geq \frac{u^+ p}{u^- (1-p)}$.

5.1.1 Overview of results we get from the above regression

Before we apply the admissible ranges analysis, it is worthwhile to consider what happens when we change the value of λ.

When λ ranges from 0.01 to a turning point (call it **T**) between 1.5 and 2, v_1 decreases from a value larger than 1 to a value less than 1, even decrease to very small value as λ increases further, while v_2 is always the smallest number it is allowed to take in the estimations, 0.01 in the table, and $v_2 - v_1$<0. In our experiments, we have no means to determine the exact value of T, but we believe that the turning point is $\lambda = \frac{u^+ p}{u^- (1-p)}$ in a perfect theoretical world. The result shows some radical risk preference behaviour of UK pension fund investors, in this area, that the agent is always extremely risk loving to loss and, as λ increases up to the turning point, the risk preference to gain changes from initially risk loving to risk neutral and then to risk averse. The local optimal estimation in this interval is λ=0.51, v_1 =0.99, v_2=0.01, $\sum_{i=1}^{n} |\varepsilon_i|$=8.55960.

When λ crosses the turning point T to some arbitrary large number, which in our experiments we tried up to λ=1000, the risk preference becomes quite different from that in the previous stage. v_1 always takes the smallest number it is allowed in the estimations, that is, 0.01 in the our experiments, while v_2 increases from value less than 1 to value larger than 1 as λ increases, and we have $v_2 - v_1$ >0. The agent is extremely risk averse to gain and, as λ increases up to larger number, the risk preference to loss changes from initially risk loving to risk neutral and then to risk averse. The local optimal estimation in this interval is λ=8.60, v_1 =0.01, v_2 =1.86, $\sum_{i=1}^{n} |\varepsilon_i|$ =10.44738 and the average absolute error is 0.36025.

5.1.2 Admissible ranges analysis on the original results

Since the investor we study in this paper is the representative UK pension funds investor, it is reasonable to assume that the proportion of wealth held by UK pension funds in risky asset is an increasing function of the probability that equity outperforms the safe asset. Then the above assumption implies that the admissible range of λ is $\lambda \geq \frac{u^+ p}{u^- (1-p)}$. So it makes sense to reject the first local optimal result at λ=0.51, as Hwang and Satchell (2003) have shown that this lower bound of λ is much larger than 1 in practice. But for a formal proof, we plug the parameters at λ=0.51 into our estimation program to evaluate $\frac{u^+ p}{u^- (1-p)}$, and we get $\frac{u^+ p}{u^- (1-p)}$ = 0.780, which violates the admissible condition of $\lambda \geq \frac{u^+ p}{u^- (1-p)}$. While, the value of $\frac{u^+ p}{u^- (1-p)}$ is 2.870 when λ=8.60; this satisfies the condition of $\lambda \geq \frac{u^+ p}{u^- (1-p)}$; we reject the local optimal result at λ=0.51 and only accept the result at λ=8.60.

To summarize the above discussion, we have:

When λ =8.60, v_1=0.01<1, v_2=1.86>1, the optimal result shows that a representative UK pension fund investor is risk averse with respect to both gain and loss. This is sketched in the Figure 1.2a and 1.2b below. Moreover, we infer from the above optimal result that the representative UK pension fund investor is extremely risk averse.

Figure 1.2a: $u\left(X\right)=\dfrac{X^{0.01}}{0.01}$ when X>0

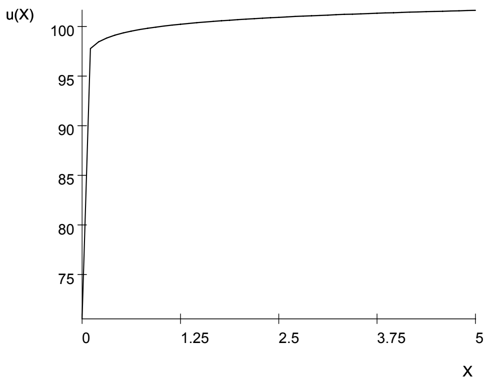
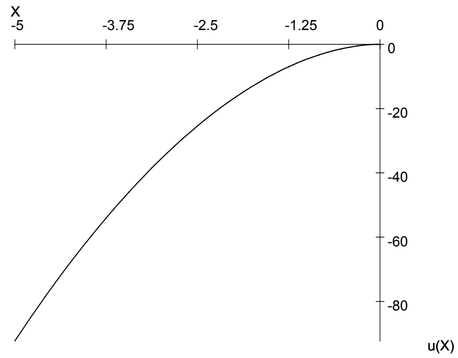


Figure 1.2b: $u\left(X\right)=-8.60\dfrac{\left(-X\right)^{1.86}}{1.86}$ when X<0



5.2 Results with 20 years UK Gilt bond as Benchmark

Although there is still the constraint of available market data when we use the UK Gilt bond as benchmark, we are better off with the data set of 36 observations (1965-2000) for this regression. The results have been listed in **Table 2 in Appendix B**. The table shows an outcome similar to that of section 5.1. Bolded numbers are two local optimal results.

5.2.1 A brief of the results with 20 years UK Gilt bond as Benchmark

The pattern of λ, v_1, v_2 and $\sum_{i=1}^{n}\left|\varepsilon_i\right|$ is quite similar to that of section 5.1 and an intuitive result can be found from the above Table 2 and Figure 2, so we focus on the comparison between the results with the two different benchmark.

The evaluation is essentially the same as previous. We reject an inadmissible local optimum and find that λ=16.33, v_1 =0.01, v_2 =3.30, $\sum_{i=1}^{n}|\varepsilon_i|$ =15.152422 and λ $> \frac{u^+ p}{u^-(1-p)} =$ 1.463115508 at λ=16.33. This result also shows that UK pension fund investors are risk averse on both side of gain and loss with 20 years UK Government Gilt as Benchmark as well. Also see the figures below.

Figure 2.2 a: $u(X) = \frac{X^{0.01}}{0.01}$ when X>0

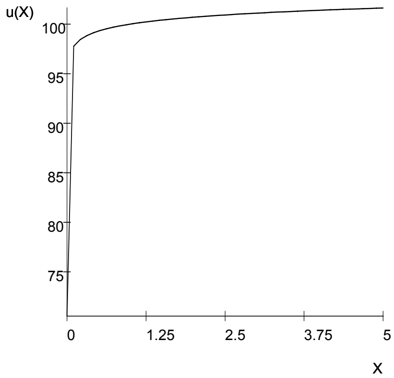
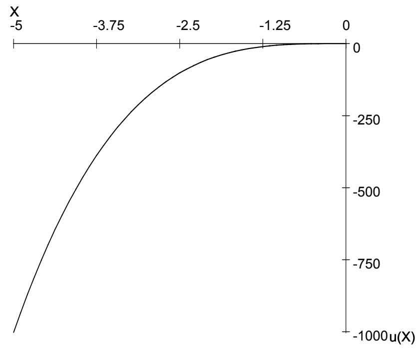


Figure 2.2b: $u(X) = -16.33\frac{(-X)^{3.30}}{3.30}$ when X<0



5.2.2 Comparing the results from two different benchmarks

Both of the results show that the UK pension fund investors are risk averse to gain and loss, especially so with respect to gain. Virtually most of the gain occurs just above the benchmark; after that the utility is almost flat. This argument seems to be consistent with the fact that in the real world the target of pension funds is not excessive profit with large potential risk but being in excess of their liabilities. These liabilities are probably captured reasonably accurately by Gilts and bills.

The values of λ and v_2 to make the estimation result converge in the Gilt Benchmark case are larger than those to the 3 months UK Government bill case. Moreover, when we use 3 months bills as benchmark, the optimal result converges at a very small range around λ=8.60, while we can achieve the minimum sum of absolute residuals with λ values from 16.33 to 20 in the case of 20 years Gilt Benchmark. Since in both cases the agent are risk averse on both sides and, in general, the rate of return from 20 years UK Government Gilt is more than that from 3 months UK Government bill, this may implies that the agent becomes more risk averse to loss, if the rate of return from the benchmark increases. This conclusion can also be put in an intuitive way that the agent feels more pain to a loss if the assumed safe return becomes higher, i.e. if the opportunity cost rises. There seems no result in the prospect literature concerning comparative static with respect to different benchmark.

5.3 Comparison with existing results in Hwang and Satchell (2003)

In this section, we see what happens if we fix λ to the value argued by Hwang and Satchell; we reproduce of their result in **table 5.3.1 and 5.3.2** below. We now estimate our models fixing λ=2.25 and λ=3, our results are presented in **table 5.3.3**. Because our regression results for v_1 always converges to 0.1 so we only show the data with the same λ and v_1 in Hwang and Satchell (2003) to compare with. We do not show the regression result at λ=1.5 either, because it is too close to the lower boundary of λ and we do not have a sensible convergent result. This also happened in Hwang and Satchell's paper. They have unacceptable results with violation of $v_2 > v_1$ under λ=1.5. Moreover, our results indicate the same risk preference with that in Hwang and Satchell (2003) under the same λ and v_1 that investors are risk averse to gain and risk loving to loss. However, we could also see that the v_2 we have in table 5.3.3 for the same λ and v_1 are larger than those in table 5.3.1 and 5.3.2, which means that the UK pension funds are less risk loving than the investors in the whole market. This is likely due to the different investors group that we investigate from that in Hwang and Satchell's paper.

Table 5.3.1(The value of v_2 for given sets of v_1 and θ)

v_1=0.1	UK market (10/1982-09/2002)				
	(with Normal distribution)				
$v_2 \backslash \theta$	0.40	0.50	0.60	0.70	0.80
v_2 |λ=1.5	**0.07**	**0.07**	**0.07**	**0.06**	**0.06**
v_2 |λ=2.25	0.15	0.15	0.15	0.16	0.16
v_2 |λ=3.00	0.21	0.21	0.22	0.22	0.23

Bolded numbers represent violation of proposition, i.e. $v_2 > v_1$

Table 5.3.2(The value of v_2 for given sets of v_1 and θ)

v_1=0.1	UK market (10/1982-09/2002)				
	(with KST(1995) distribution)				
$v_2 \backslash \theta$	0.40	0.50	0.60	0.70	0.80
v_2 |λ=1.5	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
v_2 |λ=2.25	0.19	0.20	0.20	0.21	0.21
v_2 |λ=3.00	0.26	0.27	0.28	0.29	0.30

Bolded numbers represent violation of proposition, i.e. $v_2 > v_1$

Table 5.3.3 (Results in this table by 0.1 degree of accurate for v_1 and given λ)

	λ	v_1	v_2
3M Bill Benchmark	2.25	0.1	0.38
3M Bill Benchmark	3	0.1	0.70
20yr Gilt Benchmark	2.25	0.1	0.49
20yr Gilt Benchmark	3	0.1	0.75

5.4 Further experiments and discussion

We run all the above regressions by assuming ε_i ~iid, and subject to a symmetric distribution. But from the statistics description in the second column of table 5.4.2 below, the residual does not follow a Normal distribution in both case of different benchmarks, but a very fat-tailed distribution with insignificant negative skewness and quite a few outliers. The hypothesis test of zero mean is not rejected at 95% confidence level. This does not invalidate our estimation principle which is distribution free, in fact it adds weight to our decision not to use maximum-likelihood.

However, we note that there exists problems of big outliers and a fairly large sum of absolute residuals. We are interested in doing more estimations under different conditions to improve the results. Firstly, we pick out all the outliers with absolute error larger than 0.5 and then run the regressions again without those outliers. In the second place, we split the sample into two periods and add a dummy variable for λ to test differences of λ and to see whether there is any improvement in the estimation. The results of these estimations are reported in table 5.4.1 and 5.4.2.

We divide the sample period into two parts to add in dummy variables for λ. For 3 months UK Government Bill benchmark regression, we have the sample split as 1972-1986(15 observations with the average investment proportion on risky asset θ=62%) and 1987-2000(14 observations with θ=76.79%), while for the regression with 20 year UK Gilt Bond benchmark one, we have 1965-1983(19 observations with θ=57.68%) and 1984-2000(17 observations with θ=75.76%).

Before we pick out the outliers, we expect that if the outliers in the regression with different benchmark are not the same, or vary a lot, then it may indicate there is some problem in our algorithm. But the result turns out that the outliers in the two regressions are consistent with each other, although they have different period length. For a regression with 3 months UK Government Bill benchmark, we find that Year 1983, 1987, 1993, 1995 and 1996 are outliers; while 1965, 1967, 1968, 1971, 1983, 1987, 1993, 1995 and 1996 for 20 year Gilt benchmark regression. Moreover, the sign of positive and negative are also consistent. The same results have been testified again when we compare to regressions with dummy variable for λ. To explain why these outliers happened in above years will require further study and is beyond the scope of this paper.

Table5.4.1: Estimation results of further experiments

3M	3M w Outlier	3M w/t Outlier	Dummy w Outlier	Dummy w/t Outlier
λ_1	8.60	8.83	7.6	6.3
λ_2	N/A	N/A	8.86	7.13
v_1	0.01	0.01	0.01	0.01
v_2	1.86	2.1	1.85	1.57
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	10.44738	5.29671	10.34468	5.28237
20Yr	20Yr w Outlier	20Yr w/t Outlier	Dummy w Outlier	Dummy w/t Outlier
λ_1	16.33	12.85	23.5	12.8
λ_2	N/A	N/A	13.65	11
v_1	0.01	0.01	0.01	0.01
v_2	3.30	3.14	3.32	3.12
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	15.15242	5.575118719	14.80395235	5.366922501

(where 3M=3 months UK Government Bill, 20Yr= 20 year UK Gilt Bond, w=with and w/t=without)

Table5.4.2: Statistics Description of regression residuals

3M	**3M w Outlier**	**3M w/t Outlier**	**Dummy w Outlier**	**Dummy w/t Outlier**
Mean	-0.055914314	-0.020150187	-0.079514475	-0.014184757
Std Error	0.092503487	0.056752135	0.0914892	0.05636449
Sample Var	0.248149958	0.077299317	0.242737938	0.076246938
Kurtosis	1.154654262	-0.922053184	1.377337979	-1.081813746
Skewness	-0.004396786	-0.066684359	0.133870332	-0.123148953
# of Observation	29	24	29	24
20Yr	**20Yr w Outlier**	**20Yr w/t Outlier**	**Dummy w Outlier**	**Dummy w/t Outlier**
Mean	-0.168644714	-0.024984007	-0.139012325	-0.046924435
Std Error	0.092065988	0.050557014	0.091181311	0.051095329
Sample Var	0.305141261	0.069012316	0.299305131	0.070489782
Kurtosis	0.213762912	-0.591993029	-0.008383809	-0.490473576
Skewness	-0.170783939	-0.000819801	-0.172107717	0.070208639
# of Observation	36	27	36	27

(where 3M=3 months UK Government Bill, 20Yr= 20 year UK Gilt Bond, w=with and w/t=without. The Kurtosis above is defined as the fourth order moment minus 3.)

From the above tables we infer that:

1) The sum of absolute residuals is reduced significantly in the regressions without the outliers, especially for the regression with 20 year UK Gilt Bond benchmark. This indicates that the outlier problem is more serious in this regression than the 3 months UK Government Bill benchmark one.

2) Furthermore, by removing the outliers, the residual distributions in both cases become platykurtic to the Normal distribution, and some now have the problem of skewness. Concerning the nature of our estimation, it is likely that it went better if the distribution is symmetric, so we regard the regressions without serious skewness problems as sensible estimations. Then we are inclined to consider results from experiments of 3 months UK Government Bill benchmark with outliers and 20 year UK Gilt Bond without outlier as our best estimation. That is

	λ	v_1	v_2
With 3M Bill Benchmark with outliers	8.60	0.01	1.86
With 20 yr Gilt Benchmark without outliers	12.85	0.01	3.14

3) The values of λ and v_2 to make the estimation result converge with Gilt Benchmark are still systematically larger than those of estimation with 3 months UK Government bill as Benchmark. This fact gives support to our argument in section 5.1.2 that the agent becomes more risk averse to loss, or more sensitive to loss, if the rate of return from the benchmark increases.

4) Moreover, if we look at the dummy variable test results, with 3 months UK Bill as the benchmark, the optimal result still converges at a very small range round λ=8.60, while we get a very large gap of λ_1 and λ_2 in the case of 20 years Gilt Benchmark with outliers. But this gap shrinks dramatically in the result without outliers. This fact shows again how severely the regressions with 20 years Gilt Benchmark with outliers

are effected by outliers. This is also one of the reasons we adjust our best estimation with 20 yr Gilt Benchmark to the results of the experiment without outliers.

5) The tests of trend of λ show a contradictory result with different benchmark, which is different from our anticipation before the tests. We expect $\lambda_1 > \lambda_2$, according to proposition 3 in Hwang and Satchell(2003), since the average investment proportion on equity (θ) increases through the time. However, we only find $\lambda_1 > \lambda_2$ in the case of 20 year UK Gilt Bond benchmark but $\lambda_1 < \lambda_2$ in the 3M Bill Benchmark case. We argue that there are following possible reasons for this result. Above all, the relationship between θ and λ is not a one to one mapping; θ can be affected by many factors. Secondly, the estimations of λ can also be affected by the financial market data, while we are using different samples for the two different benchmark estimations, so it is possible to have different results.

6 Conclusion

In this study we investigate the values of the LA parameter of UK pension fund investors using one period optimal asset allocation model. We first review the theoretical framework and discuss the assumptions in this paper. Then we establish our nonlinear regression equation followed by an analysis of difficulties inherent in our procedure. We show how to employ a numerical approach to tackle this problem rather than linearizing the nonlinear terms. This approach successfully identifies the best LAD estimate with the two different benchmarks. In the last section, we present a further discussion of the distribution of residuals and the trend of λ. A more robust result is finally reported. Obtained these estimates, we then infer the risk attitude of representative UK pension fund investor.

The results indicate that our target investor group, representative UK pension fund, is extremely risk averse and much more sensitive to loss than most other investors in the market. Concerning the nature of pension funds, our results give a positive affirmation to their investment strategy. This also shows a contradiction to the traditional experimental results on agent's risk preference; although this contradiction may well give rise to different sample agents we are looking at. Moreover, all estimated parameters fall into the admissible range and confirm the propositions derived in Hwang and Satchell's paper. In addition, we tested the effect of choosing different benchmarks and found that the agent would feel more painful to loss if the opportunity cost went up encapsulated by a rise in benchmark. This are paralleled with the high performance of equity indices in the late 90's and the more to index fund from active management. By comparing the results in similar conditions in Hwang and Satchell(2003), we found support for our estimates and also analyzed why our best estimations results are different from theirs.

Bibliography:

Amemiya, T (1982). The Two-Stage Least Absolute Deviation Estimators, Econometrica, Vol. 50 (1-3), pp. 689-711

Barberis, N., M.Huang and T. Santos (2001) "Prospect Theory and Asset Prices," Quarterly Journal of Economics 116(1), 1-53

Berkelaar, A. and R. Kouwenberg (2000) "Optimal Portfolio Choice Under Loss Aversion," Econmetric Institute Report EI2000-08/A, Erasmus University Rotterdam, Netherlands.

Berkelaar, A. and R. Kouwenberg (2000) "From Boom til Bust: How Loss Aversion Affects Asset Prices," Econmetric Institute Report EI2000-21/A, Erasmus University Rotterdam, Netherlands.

Blake. D., Lehmann, B. N. and Timmermann A. (1999) "Asset Allocation Dynamics and Pension Fund Performance", Journal of Business, Vol.72, No.4, 429-461

Dasgupta. M, and Mishra. SK. (2004) "Least Absolute Deviation Estimation of Linear Econometric Models: A Literature Review", North Eastern Hill University Working Paper Series.

Hwang. S., and Satchell. S. (2003) "The Magnitude of Loss Aversion Parameters in Financial Markets"

Kahneman, D. and A. Tversky (1979) "Prospect Theory: An Analysis of Decision Under Risk", Econometica, 47, 263-291.

Kahneman, D. and A. Tversky (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty", Journal of Risk and Uncertainty 5, 297-323

Knight, J. L., S. E. Satchell, and K. C. Tran (1995) "Statistical Modelling of Asymmetric Risk in Asset Return," Applied Mathematical Finance 2, 155-172

Markowitz, H. (1952) "Portfolio Selection," Journal of Finance 7(1), 77-91

National Statistics, WM

Powell, J L (1984). Least Absolute Deviations Estimation for the Censored Regression Model. Journal of Econometrics, Vol. 25, pp974-992

UBS Pension Fund Indicators: A long-term perspective on pension fund investment, release June 2004

Appendix A:

Figure Average pension fund – distribution of assets by investment sector (%)

End	UK Eq	Os Eq	UKB	I- L	OSB	Cash	Prop
1962	47	–	51	–	–	2	–
1963	47	–	51	–	–	2	–
1964	46	–	50	–	–	2	2
1965	47	–	48	–	–	3	2
1966	43	1	49	–	–	2	5
1967	47	1	45	–	–	2	5
1968	54	1	36	–	–	3	6
1969	52	1	36	–	–	3	8
1970	50	2	34	–	–	4	10
1971	56	2	31	–	–	3	8
1972	57	4	25	–	–	5	9
1973	48	4	26	–	–	8	14
1974	34	4	27	–	–	16	19
1975	45	5	26	–	–	9	15
1976	44	5	28	–	–	7	16
1977	45	4	28	–	–	6	17
1978	45	5	28	–	–	6	16
1979	45	5	26	–	–	7	17
1980	46	8	25	–	–	4	17
1981	45	10	21	2	–	4	18
1982	44	12	22	3	–	4	15
1983	45	15	20	3	–	4	13
1984	49	14	17	3	1	4	12
1985	51	14	17	3	1	3	11
1986	53	16	14	3	1	4	9
1987	54	13	14	3	1	5	10
1988	52	16	12	3	1	6	10
1989	52	20	8	3	2	6	9
1990	52	18	8	3	2	7	10
1991	55	20	7	3	3	4	8
1992	56	21	6	3	3	4	7
1993	57	24	4	3	3	4	5
1994	54	23	5	4	4	4	6
1995	55	22	6	5	3	4	5
1996	53	22	6	5	3	6	5
1997	53	20	7	5	3	7	5
1998	51	20	9	6	4	5	5
1999	51	24	9	4	4	4	4
2000	49	22	12	5	4	5	3

Source: National Statistics, WM

UK Eq = UK Equities
OS Eq = Overseas Equities
UKB = UK Bonds
I-L = Index-linked Gilts
OSB = Overseas Bonds
Cash = Cash
Prop = Property

Appendix B

Table 1: Results with 3 months UK Government bill as Benchmark

λ	**0.51**	1.5	2	2.25	3	4
v_1	**0.99**	0.07	0.01	0.01	0.01	0.01
v_2	**0.01**	0.01	0.15	0.23	0.52	0.87
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	**8.55960**	10.55610	14.12853	13.14464	11.98726	11.28983
λ	5	6	7	8	**8.60**	9
v_1	0.01	0.01	0.01	0.01	**0.01**	0.01
v_2	1.13	1.41	1.60	1.77	**1.86**	1.94
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	10.89129	10.65761	10.50614	10.45521	**10.44738**	10.45296
λ	10	11	12	15	20	50
v_1	0.01	0.01	0.01	0.01	0.01	0.01
v_2	2.16	2.36	2.58	2.95	3.42	4.96
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	10.46527	10.47832	10.50093	10.61002	10.74659	11.08003

Figure 1: Plotted by the data in table 1 (Where **E**= $\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$)

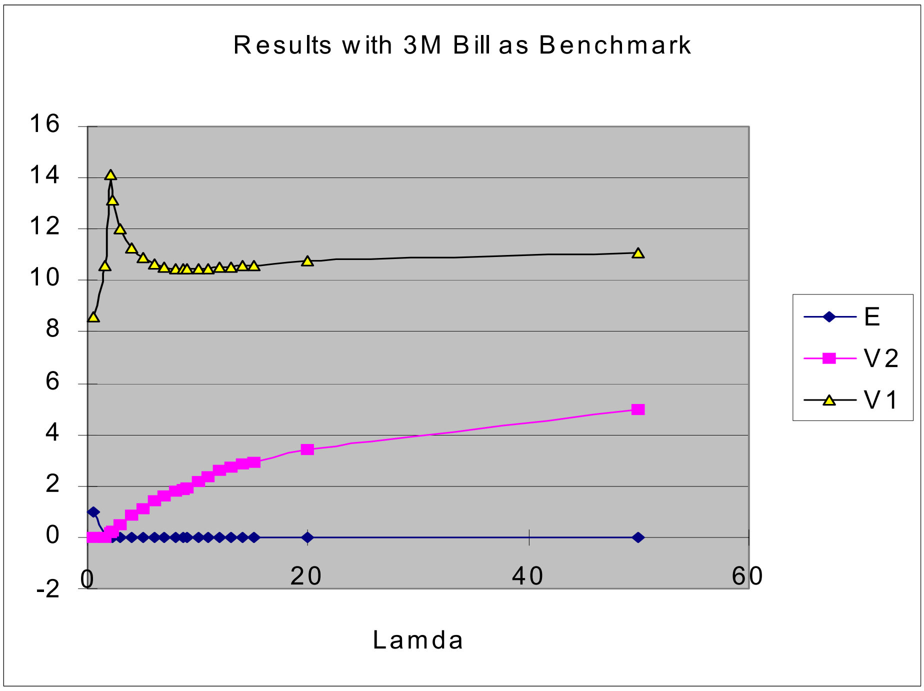


Table 2: Results with 20 years UK Government Gilt as Benchmark

λ	**0.36**	2	2.25	3	4	5
v_1	**1.21**	0.01	0.01	0.01	0.01	0.01
v_2	**0.01**	0.21	0.30	0.58	0.85	1.17
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	**12.26707**	18.65615	17.90980	16.80621	16.11167	15.66012
λ	6	7	8	9	10	11
v_1	0.01	0.01	0.01	0.01	0.01	0.01
v_2	1.41	1.63	1.79	1.96	2.17	2.37
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	15.45726	15.34125	15.26563	15.21455	15.19433	15.17932
λ	12	13	14	15	16	**16.33**
v_1	0.01	0.01	0.01	0.01	0.01	**0.01**
v_2	2.56	2.74	2.92	3.08	3.25	**3.30**
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	15.16856	15.16117	15.15630	15.15362	15.15247	**15.15242**
λ	17	19	20	25	30	50
v_1	0.01	0.01	0.01	0.01	0.01	0.01
v_2	3.30	3.30	3.30	3.43	3.74	4.62
$\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$	15.15242	15.15242	15.15242	15.15284	15.15660	15.18144

Bolded numbers are two local optimal results.

Figure 2: Plotted by the data in table 2 (Where **E**= $\sum_{i=1}^{n}\lvert\varepsilon_i\rvert$)